# GATE 2026 TEST SERIES

## Control Systems

## Test 2

1. The open loop transfer function of a unity feedback system is given by,

$G(s) = \dfrac{K}{s(1+sT_1)(1+sT_2)}$. The correct expression for gain K in terms of

$T_1$, $T_2$ and specified gain margin, $K_g$.

A ) The expression for gain K in terms of $K_g$, $T_1$ and $T_2$ is,

$K = K_g (T_1 + T_2)$

B ) The expression for gain K in terms of $K_g$, $T_1$ and $T_2$ is,

$K = K_g \left( \dfrac{1}{T_1} + \dfrac{1}{T_2} \right)$

C ) The expression for gain K in terms of $K_g$, $T_1$ and $T_2$ is,

$K = \dfrac{1}{K_g} (T_1 + T_2)$

D ) The expression for gain K in terms of $K_g$, $T_1$ and $T_2$ is,

$K = \dfrac{1}{K_g} \left( \dfrac{1}{T_1} + \dfrac{1}{T_2} \right)$

Correct Answer: **D** *Not Attempted :* ! Marks : 2 / 0.66 (-Ve)

Given that, $G(s)=\dfrac{K}{s(1+sT_1)(1+sT_2)}$

Let $s=j\omega$,

$$G(j\omega)=\frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}=\frac{K}{j\omega(1+j\omega T_2+j\omega T_1-\omega^2T_1T_2)}$$

$$=\frac{K}{j\omega[1+j\omega(T_1+T_2)-\omega^2T_1T_2]}=\frac{K}{j\omega-\omega^2(T_1+T_2)-j\omega^3T_1T_2}$$

$$=\frac{K}{-\omega^2(T_1+T_2)+j\omega(1-\omega^2T_1T_2)}$$

The gain margin, $K_g$ is defined as the reciprocal of the magnitude of $G(j\omega)$ at phase crossover frequency. At phase crossover frequency the magnitude is purely real. Hence at phase crossover frequency $\omega_{pc}$, the imaginary part of $G(j\omega)$ is zero.

$\therefore$ At $\omega=\omega_{pc}$, $\quad \omega_{pc}(1-\omega_{pc}^2T_1T_2)=0 \Rightarrow 1-\omega_{pc}^2T_1T_2=0$

$$\Rightarrow -\omega_{pc}^2T_1T_2=-1$$

$$\therefore\ \omega_{pc}=\frac{1}{\sqrt{T_1T_2}}$$

At $\omega=\omega_{pc}$, the imaginary part is zero,

$$\therefore\ |G(j\omega)|=\left|\frac{K}{-\omega^2(T_1+T_2)}\right|=\frac{K}{\omega^2(T_1+T_2)}$$

$$\therefore \text{ Gain margin, } K_g=\frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}=\frac{1}{K/\omega_{pc}^2(T_1+T_2)}=\frac{\omega_{pc}^2(T_1+T_2)}{K}$$

Put, $\omega_{pc}^2=\dfrac{1}{T_1T_2}$ in the above equation

$$\therefore\qquad K_g=\frac{\left(\frac{1}{T_1T_2}\right)(T_1+T_2)}{K}\Rightarrow K=\frac{1}{K_g}\frac{T_1+T_2}{T_1T_2}=\frac{1}{K_g}\left(\frac{T_1}{T_1T_2}+\frac{T_2}{T_1T_2}\right)\Rightarrow$$

$$K=\frac{1}{K_g}\left(\frac{1}{T_1}+\frac{1}{T_2}\right)$$

2. The transfer function of a system is:

$$\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)}$$

In the state-space representation of the system, the minimum number of state variables necessary is

A ) 2

B ) 3

C ) 4

D ) 1

Correct Answer: **B** *Not Attempted :* Marks : 1 / 0.33 (-Ve)

For 3rd order system, 3 state variables are required.

3. The transfer functions of two compensators are given below:

$$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

A) $C_1$ is a lead compensator

B) $C_2$ is a lag compensator

C) $C_1$ is a lag compensator

D) $C_2$ is a lead compensator

Correct Answer: **A,B** *Not Attempted :* ! Marks : 2 / 0 (-Ve)

$$\angle C_1 = \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{10} \text{ is always positive}$$

$$\angle C_2 = \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{1} \text{ is always negative}$$

$\therefore C_1$ is lead & $C_2$ is lag compensator.

4.

The open loop transfer function, $G(s) = \frac{1}{s(1+2s)(1+s)}$ has

A ) The phase crossover frequency, $\omega_{pc} = 0.707$ rad /sec

B ) The phase crossover frequency, $\omega_{pc} = 0.577$ rad /sec

C ) The gain margin in db = 3.5 db

D ) The gain margin in db = 2.5 db

Correct Answer: **A,C** *Not Attempted :* ! Marks : 2 / 0 (-Ve)

To find phase crossover frequency and gain margin

Given that, $G(s)=\frac{1}{s(1+2s)(1+s)}$

Let $s=j\omega$

$$\therefore\ G(j\omega)=\frac{1}{j\omega(1+j2\omega)(1+j\omega)}$$

$$=\frac{1}{j\omega(1+j\omega+j2\omega-2\omega^2)}$$

$$=\frac{1}{j\omega(j3\omega+1-2\omega^2)}$$

$$=\frac{1}{-3\omega^2+j\omega(1-2\omega^2)}$$

At phase crossover frequency the imaginary part of $G(j\omega)$ is zero.

Hence put $\omega=\omega_{pc}$ in imaginary part and equate to zero to solve for $\omega_{pc}$.

$$\therefore\ \omega_{pc}\left(1-2\omega_{pc}^2\right)=0$$

Since $\omega_{pc}\neq 0,\ 1-2\omega_{pc}^2=0 \Rightarrow -2\omega_{pc}^2=-1 \Rightarrow \omega_{pc}^2=\frac{1}{2}$

$$\Rightarrow\ \omega_{pc}\frac{1}{\sqrt{2}}=0.707\ \text{rad/sec}$$

The gain margin, $K_g$ is defined as reciprocal of magnitude of $G(j\omega)$ at phase cross over frequency.

Gain margin,

$$K_g=\frac{1}{\left|G(j\omega)\right|_{\omega=\omega_{pc}}}=\frac{1}{\left|1/-3\omega^2\right|_{\omega=\omega_{pc}}}=3\omega_{pc}^2=3\times 0.707^2=1.5$$

Gain margin in db = $20\log K_g = 20\log 1.5 = 3.5$ db

5. The transfer function of a basic PD controller is given by (K's are real constants)

A ) $K_0 + \frac{K_1}{s} + K_2 s$

B ) $K_1 s + K_2 s$

C ) $K_0 + K_2 s$

D ) $K_0 + \frac{K_1}{s}$

Correct Answer: **C** *Not Attempted :* Marks : 1 / 0.33 (-Ve)

The PD controller is described by $y(t) = K_0 x(t) + K_2 \frac{dx}{dt}$

$$\therefore \frac{Y(s)}{X(s)} = G(s) = K_0 + K_2 s$$

6.

The open loop transfer function of a system is $G(s)=\frac{K}{s(1+0.1s)(1+s)}$.

A ) The value of $K=1.837$ so that phase margin is $40°$

B ) The value of $K=1.378$ so that phase margin is $40°$

C ) The value of $K=1.873$ so that phase margin is $40°$

D ) The value of $K=1.738$ so that phase margin is $40°$

Correct Answer: **B** *Not Attempted :* ! Marks : 2 / 0.66 (-Ve)

To find K for specified phase margin:

Given that, $G(s)=\dfrac{K}{s(1+0.1s)(1+s)}$, Let $s=j\omega$

$\therefore$

$$G(j\omega)=\frac{K}{j\omega(1+j0.1\omega)(1+j\omega)}=\frac{K}{\omega\angle 90^\circ\sqrt{1+(0.1\omega)^2}\angle\tan^{-1}0.1\omega\sqrt{1+\omega^2}\angle\tan^{-1}\omega}$$

$$|G(j\omega)|=\frac{K}{\omega\sqrt{1+0.01\omega^2}\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)=-90^\circ-\tan^{-1}0.1\omega-\tan^{-1}\omega$$

Let, $\omega_{gc}$ = Gain crossover frequency

$\phi_{gc}=\angle G(j\omega)$ at $\omega=\omega_{gc}$

At $\omega=\omega_{gc}$, $\phi_{gc}=\angle G(j\omega)\big|_{\omega=\omega_{gc}}=-90-\tan^{-1}0.1\omega_{gc}-\tan^{-1}\omega_{gc}$

By definition of phase margin,

Phase margin, $\gamma=180^\circ+\phi_{gc}$

The required phase margin is 40°, $\therefore\gamma=40^\circ$

$$\therefore 40^\circ=180^\circ-90^\circ-\tan^{-1}0.1\omega_{gc}-\tan^{-1}\omega_{gc}$$

$$\Rightarrow\tan^{-1}0.1\omega_{gc}+\tan^{-1}\omega_{gc}=180^\circ-90^\circ-40^\circ$$

$$\therefore\tan^{-1}0.1\omega_{gc}+\tan^{-1}\omega_{gc}=50^\circ$$

On taking tan on either side we get,

$$\tan[\tan^{-1}0.1\omega_{gc}+\tan^{-1}\omega_{gc}]=\tan 50^\circ$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\ \tan B}$$

$$\frac{\tan\ \tan^{-1}0.1\omega_{gc}+\tan\ \tan^{-1}\omega_{gc}}{1-\tan\ \tan^{-1}0.1\omega_{gc}\times\tan\ \tan^{-1}\omega_{gc}}=\tan 50^\circ$$

$$\Rightarrow\frac{0.1\omega_{gc}+\omega_{gc}}{1-0.1\omega_{gc}\times\omega_{gc}}=1.192\Rightarrow\frac{1.1\omega_{gc}}{1-001\omega_{gc}^2}=1.192$$

On cross multiplying the above equation we get,

$$1.1\omega_{gc} = 1.192\left(1 - 0.1\omega_{gc}^2\right) \Rightarrow \quad 0.1192\omega_{gc}^2 + 1.1\omega_{gc} - 1.192 = 0$$

$$\therefore\ \omega_{gc}^2 + \frac{1.1}{0.1192}\omega_{gc} - \frac{1.192}{0.1192} = 0 \Rightarrow \omega_{gc}^2 + 9.228\omega_{gc} - 10 = 0$$

$$\therefore\ \omega_{gc} = \frac{-9.228 \pm \sqrt{9.228^2 + 4\times 10}}{2}$$

$$= \frac{-9.228 \pm 11.1873}{2}$$

On taking positive value we get,

$$\omega_{gc} = \frac{-9.228 + 11.1873}{2} = 0.98\ \text{rad/sec}$$

At $\omega = \omega_{gc}$, $|G(j\omega)| = 1$ ; $\left|G(j\omega)\right|_{\omega=\omega_{gc}} = \dfrac{K}{\omega_{gc}\sqrt{1+0.1\omega_{gc}{}^2}\sqrt{1+\omega_{gc}{}^2}} = 1$

$$\therefore\ K = \omega_{gc}\sqrt{1+0.01\omega_{gc}^2}\sqrt{1+\omega_{gc}^2} = 0.98\sqrt{1+0.01\times 0.98^2}\sqrt{1+0.98^2}$$

For a phase margin of 40°, K = 1.3787.

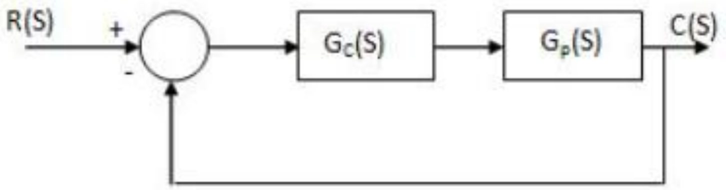
**7.** Consider a transport lag process with a transfer function $G_p(S) = e^{-s}$.

The process is controlled by a purely integral controller with transfer function $G_c(S) = \frac{K_i}{S}$ in a unity feedback configuration. The value of $K_i$ for which the closed loop plant has a pole at $s = -1$, is ________.

Correct Answer: **0.36** Answer Range: **(0.35 , 0.38)** *Not Attempted :* Marks : 2 / 0 (-Ve)

$$G_p(s) = e^{-s}$$

$$G_{c(S)} = \frac{K_i}{s}$$




Characteristics equation $1 + G(s)\,H(s) = 0$

$$\Rightarrow 1 + \frac{K_i}{S} \times e^{-s} = 0$$

$$\Rightarrow K_i = \frac{-S}{e^{-s}}$$

$$K\big|_{s=-1} = \frac{1}{e^{1}} = 1 \times e^{-1} = 0.36$$

**8.** Consider the transfer function $G(s)=\frac{2}{(s+1)(s+2)}$. The phase margin of $G(s)$ in degrees is ____.

Correct Answer: **180** Answer Range: **(180 , 180)** *Not Attempted :* Marks : 1 / 0 (-Ve)

$$P.M = 180^\circ + \angle G(s)H(s)\big|_{\omega=\omega_{gc}} \text{ ------- } (1)$$

$$\text{GCOF } (\omega_{gc}) : M\big|_{\omega=\omega_{gc}} = 1$$

$$\left|\frac{2}{(s+1)(s+2)}\right|_{\omega=\omega_{gc}} = 1$$

$$\frac{2}{\sqrt{1+\omega_{gc}^2}\,\sqrt{4+\omega_{gc}^2}} = 1$$

$$\sqrt{1+\omega_{gc}^2}\,\sqrt{4+\omega_{gc}^2} = 2$$

Square on both side

$$(1+\omega_{gc}^2)(4+\omega_{gc}^2) = 4\,;\ \text{Let } \omega_{gc}^2 = x$$

$$(x+1)\,(x+4) = 4$$

$$x^2 + 5x + 4 = 4 \Rightarrow x^2 + 5x = 0$$

$$x\,[x+5] = 0$$

$$x = \omega_{gc}^2 = 0 \text{ (or) } x = \omega_{gc}^2 = -5 \text{ [invalid]}$$

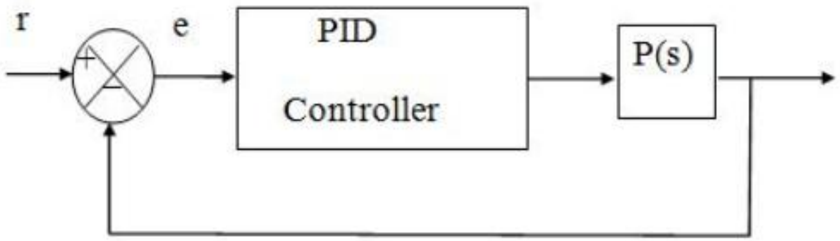
$$\omega_{gc} = 0$$

Now $\angle G(s)H(s)\big|_{\omega_{gc}=0} = 0$

Substitute in equation (1)

$$P.M = 180 + 0 = 180^\circ$$

**9.** In the control system shown in the figure below, a reference signal $r(t) = t^2$ is applied at time $t = 0$. The control system employs a PID controller $C(s) = K_p + K_I/s + K_D s$ and the plant has a transfer function $P(s) = 3/s$. If $K_p = 10$, $K_I = 1$ and $K_D = 2$, the steady state value of e is



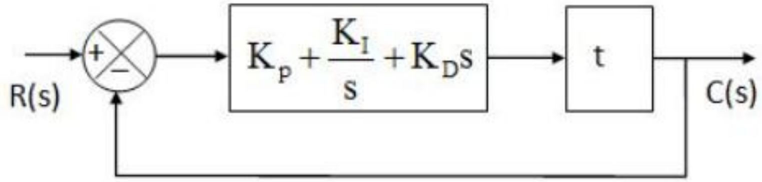

A ) 2/3

B ) 0

C ) 1

D ) ∞

Correct Answer: **A** *Not Attempted :* Marks : 2 / 0.66 (-Ve)




$$G(s) = \left(10 + \frac{1}{s} + 2s\right)\left(\frac{3}{s}\right)$$

$$G(s) = \frac{3(2s^2 + 10s + 1)}{s^2}$$

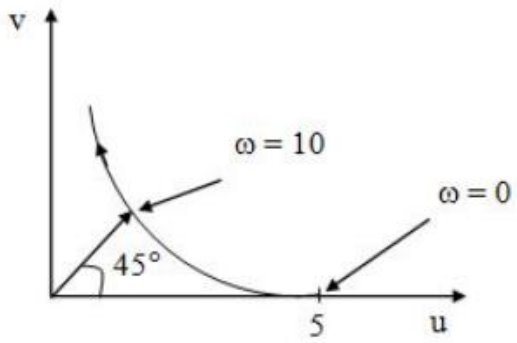
$$r(t) = t^2$$

Steady state error $e_{ss}(t^2) = \left(\frac{2}{K_a}\right)$

$$K_a = \underset{s \to 0}{\mathrm{Lt}}\, s^2 G(s) = \underset{s \to 0}{\mathrm{Lt}}\, s^2 \left[\frac{3(2s^2 + 10s + 1)}{s^2}\right] = 3$$

$$e_{ss} = \frac{2}{K_a} = \frac{2}{3}$$

**10.** The below figure shows the polar plot of a system. The transfer function of the system is




A ) $5\,(1 + 0.1\,s)$

B ) $(1 + 0.5\,s)$

C ) $5(1 + 10s)$

D ) $5\,(1 + s)$

Correct Answer: **A** *Not Attempted :* Marks : 2 / 0.66 (-Ve)

: From the plot, $G(j0) = 5$,

$$G(j\infty) = \infty \angle 90^\circ$$

$$\angle G(j10) = 45^\circ$$

$$\therefore\ G(s) = K(1 + \tau s)\ ,\ K = 5$$

$$\tan^{-1}(10\,\tau) = 45^\circ$$

$$10\,\tau = \tan 45^\circ = 1,\quad \tau = 0.1$$

$$G(s) = 5\,(1 + 0.1\,s)$$

11. The open loop part of a unity feedback control system G(s) is unstable and has two poles on the right hand side of the s-plane. However, the closed loop system is stable. The number of encirclements made by the Nyquist plot of the $(-1, 0)$ point in the G(s) plane is

A ) $2$

B ) $0$

C ) $-3$

D ) $-1$

Correct Answer: **A** *Not Attempted :* Marks : 1 / 0.33 (-Ve)

$N = Z - P$

Where

N = Number of encirclements of the Nyquist plot of G(s) around the $(-1, j0)$ point.

Z = Number of poles of the CL system in the RH of s-plane.

P = Number of poles of OLTF in the right half of s-plane = 2

As the closed loop system is stable,

$Z = 0$

$\therefore N = -P = -2$

Negative sign indicates that the encirclements are two in the anti clock wise direction if the Nyquist closed path taken in the s-plane is clockwise.

**12.** The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u,$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Where y is the output and u is the input. The system is controllable for

A ) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

B ) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

C ) $a_1 = 0, a_2 \neq 0, a_3 = 0$

D ) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

Correct Answer: **D** *Not Attempted :* ! Marks : 2 / 0.66 (-Ve)

For the given state variable description

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = [1 \ 0 \ 0]$$

$$AB = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix}, A^2B = \begin{bmatrix} a_1 a_2 \\ 0 \\ 0 \end{bmatrix}$$

Controllability matrix

$$S = [B : AB : A^2B] = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$|S| = (-a_1 a_2^2) \neq 0$

if $a_1 \neq 0$ and $a_2 \neq 0$

For Controllability, S should be nonsingular or Rank of S = 3 or $S \neq 0$

$|S| = -a_1 a_2^2 \neq 0$, when $a_1 \neq 0$ and $a_2 \neq 0$

The value of $a_3$ is immaterial.

13. The forward path transfer function L(s) of the control system shown in Figure (a) has the asymptotic Bode plot shown in figure (b). If the disturbance d(t) is given by $d(t) = 0.1\sin(\omega t)$ where $\omega = 5$ rad/s, the steady-state amplitude of the output y(t) is

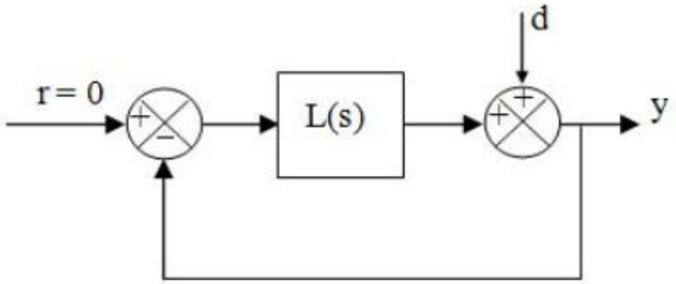



Figure (a)

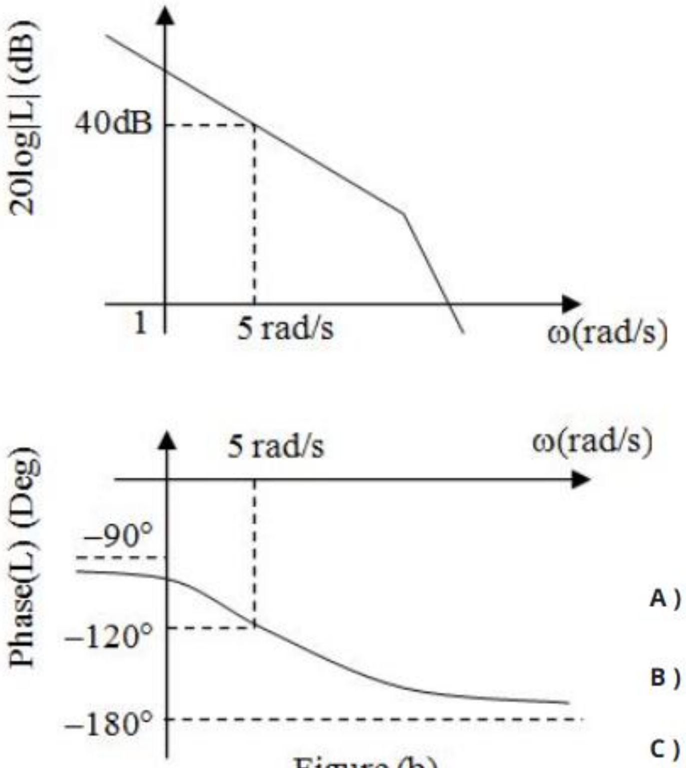
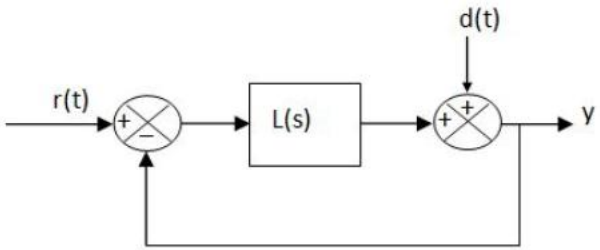



Figure (b)




$$y = \frac{rL + d}{1+L}$$

$$\frac{y}{d} = \frac{1}{1+L}$$

$$\Rightarrow y = \frac{d}{1+L} = \frac{0.1}{1+100} \approx 0.1\times10^{-2} = 10^{-3}$$

A ) $1.00 \times 10^{-3}$

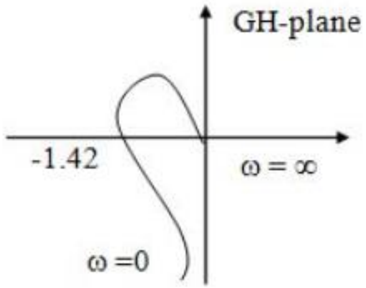
B ) $2.50 \times 10^{-3}$

C ) $5.00 \times 10^{-3}$

D ) $10.00 \times 10^{-3}$

Correct Answer: **A**    *Not Attempted :*    Marks : 2 / 0.66 (-Ve)

**14.** The polar plot of a type-1, 3-pole, open-loop system is shown in Figure below. The closed-loop system is




A ) Always stable

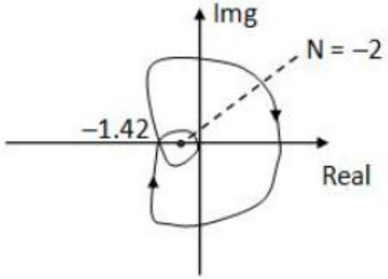
B ) Marginally stable

C ) Unstable with one pole on the right half s-plane

D ) Unstable with two poles on the right half s-plane.

Correct Answer: **D** *Not Attempted :* Marks : 1 / 0.33 (-Ve)

Nyquist plot for the given system is




Encirclement of $(-1, j0)$ $N = -2$

No. of right side poles of open loop system $P = 0$

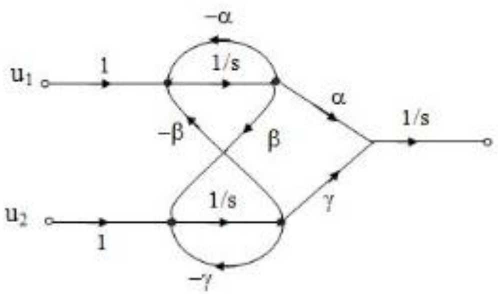
$N = P - Z$

$Z = P - N$

$= 0 - (-2) = 2$

$Z = 2$ implies, closed loop system is unstable with two right hand poles.

**15.** A signal flow graph of a system is given below.




The set of equations that correspond to this signal flow graph is

A) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}\beta & -\gamma & 0\\ \gamma & \alpha & 0\\ -\alpha & -\beta & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 0\\0 & 1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

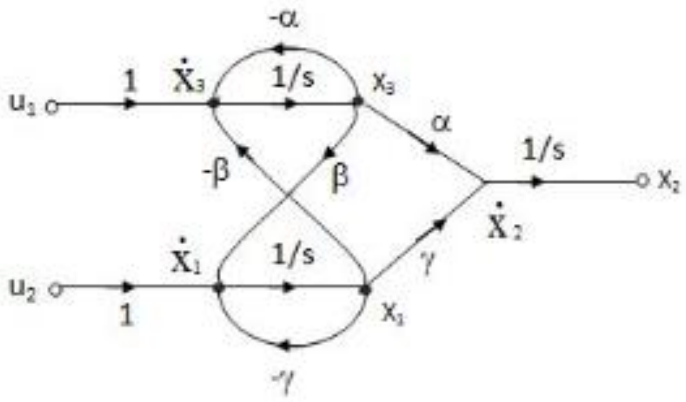
B) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0 & \alpha & \gamma\\ 0 & -\alpha & -\gamma\\ 0 & \beta & -\beta\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 1\\1 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

C) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-\alpha & \beta & 0\\ -\beta & -\gamma & 0\\ \alpha & \gamma & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

D) $\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-\gamma & 0 & \beta\\ \gamma & 0 & \alpha\\ -\beta & 0 & -\alpha\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0 & 1\\0 & 0\\1 & 0\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}$

Correct Answer: **D** *Not Attempted :* Marks : 2 / 0.66 (-Ve)

: Given signal flow graph is




$$\dot{x}_1 = -\gamma x_1 + \beta x_3 + u_2$$

$$\dot{x}_2 = \gamma x_1 + \alpha x_3$$

$$\dot{x}_3 = -\beta x_1 - \alpha x_3 + u_1$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}-\gamma & 0 & \beta\\ \gamma & 0 & \alpha\\ -\beta & 0 & -\alpha\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}0 & 1\\0 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$